Developing a concrete-pictorial-abstract model for negative number arithmetic (2016)

Context
Negative number arithmetic is frequently cited by both students and teachers as being challenging to learn, and challenging to teach. Visnovsky and Botelho (2008) found that the multiple roles of the negative sign, as a unary, binary and positional operator, presents a fundamental challenge for learners. Teaching conceptual change arises from the fact that while the magnitude of positive numbers can be visualised in terms of the cardinality of sets, negative numbers cannot. The challenge for this study was to effectively address these four conceptual features in the instructional material. According to Alexiou and Wiliam (2005), students will “instantly” invert rules to explain the patterns they see around them. This may explain why negative number arithmetic is a topic fraught with misconceptions: if the instructional approach does not address conceptual understanding, students will actively seek out rules and justifications which may not necessarily be conceptually sound. In 2012 the UK Department for Education found that a common feature of successful mathematics curricula is a greater emphasis on understanding the CPA approach (CPA = Concrete, Pictorial, Abstract). In response to this, the concrete-pictorial-abstract (CPA) sequence is an instructional approach practised in Singapore since the 1980s, and has recently gained popularity in the UK as an approach to promote conceptual understanding. It involves development of conceptual understanding by first exploring a concept through the use of concrete manipulatives; next representing the concept pictorially; and finally representing the concept symbolically. In UK secondary schools, this approach differs from conventional practice in its use of the concrete and pictorial stages. Wheat (2005) learners. A t-test revealed that the majority of students in the trial group had a significantly greater number of marks than the majority of students in the control group.

Methodology
The participating students (n = 122) were aged between 14 and 15 years, with the trial group (n = 7) and control group (n = 5) forming two existing classes which were both ranked fifth between 14 and 15 years, with the trial group (n = 7) and control group (n = 5) forming two existing classes which were both ranked fifth.

Aim of this study
This study aims to develop a CPA model for negative number arithmetic, using the number line and ‘number bars’ as the key representations. The design attempts to effectively address the four conceptual features of unary, binary, and positional operators, and magnitudes.

Qualitative evidence
Both trial and control groups completed a pre-assessment to survey the topic of negative number arithmetic and a post-assessment afterwards. The changes in students’ scores from pre- to post-assessment were compared in six areas:

Results
A comparison of the changes in scores from pre-assessment to post-assessment in the areas of ordering and division reveals little or no difference between the trial and control groups. The class teacher noted that there was a difference in change in score from the fact that while the magnitude of positive numbers can be visualised in terms of the cardinality of sets, negative numbers cannot. The challenge for this study was to effectively address these four conceptual features in the instructional material.

Research questions
Is there a difference in change in score from the fact that while the magnitude of positive numbers can be visualised in terms of the cardinality of sets, negative numbers cannot. The challenge for this study was to effectively address these four conceptual features in the instructional material.

A t-test revealed that the majority of students in the trial group had a significantly greater number of marks than the majority of students in the control group.

Discussion
The results of this study suggest that the majority of students who were taught negative number concepts using a CPA approach increased their scores in post-assessments by a significantly greater number of marks than students who were taught using a non-CPA approach in the areas of addition and subtraction with negative numbers. There is also evidence to suggest that the CPA model had a positive effect on student confidence, independence, and engagement.

Conclusion
Students with Math Difficulties in Inclusive Education (2003) noted that this level of independence was not demonstrated in students who were taught using a non-CPA approach. The class teacher noted that the vocabulary of ‘negative as opposite’ was effective in promoting conceptual understanding. This study raises the question of which concept may have been taking place.

The consistent use of vocabulary and notation became an unexpected motivator for learning, with students enjoying the opportunity to correct each other’s use of the word ‘minus’ instead of ‘negative’, and to suggest brackets should be written in order to make expressions easier to understand.